

LBYCV2E – THEORY OF STRUCTURES LABORATORY
STAREX NO. 3A & 3B
Bending Moment and Shear Due to a Moving Concentrated Load

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I. Objectives of the Experiment

- To examine the effect of a moving load on the bending moment and shear force for both magnitude and direction at the cut of a beam by applying a constant load P at various positions from the left end ($x = 0$ mm) to the right end ($x = 580$ mm).
- To identify the location of P that produces the maximum positive and negative bending moment and shear force at the cut of the presented structural model.
- To validate the theorems on influence lines for the bending moment and shear force.

II. Procedure

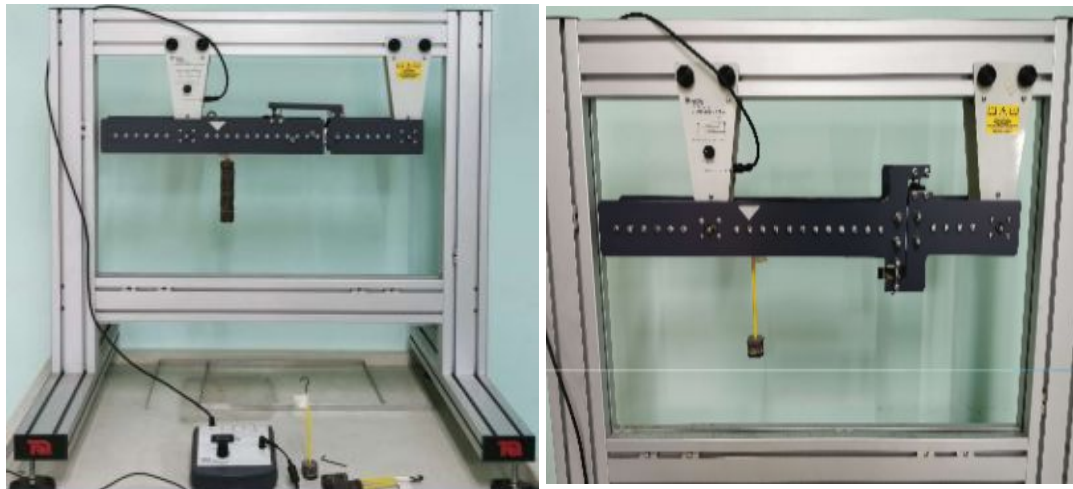


Figure 1. STR-2 Bending Moment Apparatus (left) and STR-3 Shear Force Apparatus (right)

The previous experimental setups were utilized in this experiment, except that the researchers determined the bending moment and shear force due to a moving concentrated load.

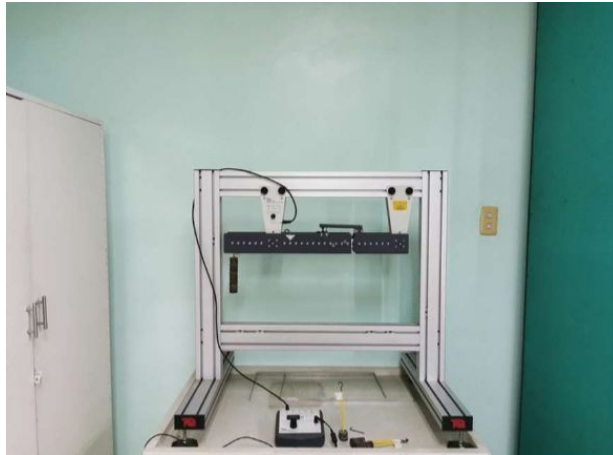


Figure 2. Bending moment apparatus with assigned load at the left

The assigned mass was applied along the beam of the bending moment apparatus from left to right, starting at $x = 0$ mm to $x = 580$ mm.

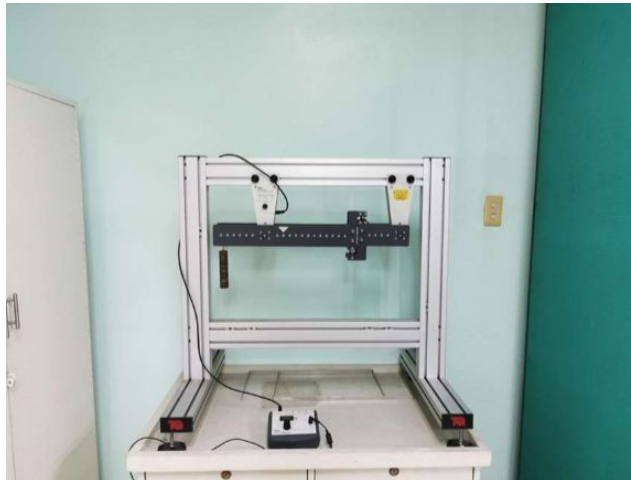


Figure 3. Shear force apparatus with assigned load at the left

The assigned mass was applied along the beam of the shear force apparatus from left to right, starting at $x = 0$ mm to $x = 580$ mm.

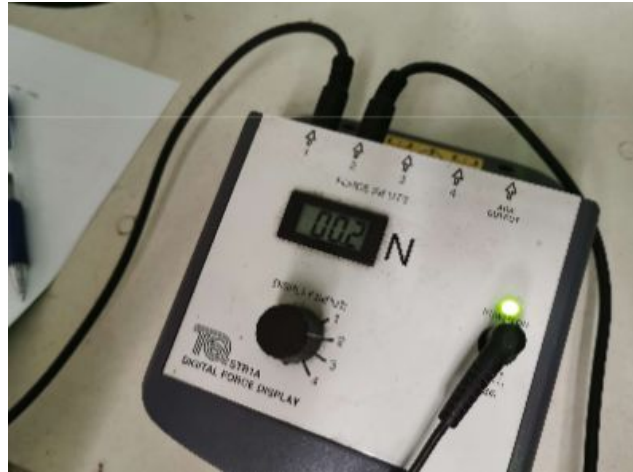


Figure 4. Reading of the force sensor

The researchers obtained a reading from the force sensor which was utilized to compare the computed bending moment or shear for each load applied. To perform this, the force sensor was initially set to 0 to attain consistency of the collated data wherein the data displayed in the force sensor as seen in *Figure 4* was recorded throughout each length interval.

III. Data and Results

Table 1. Given Data with Experimental Moment & Reactions at A and B

X	Description	P (N)	Force Data Average (N)	M _{expt} (N-mm)	R _a	R _b
0		3.93	-1.43	-178.75	5.180454545	-1.250454545
20		3.93	-1.25	-156.25	5.001818182	-1.071818182
40		3.93	-1.00	-125	4.823181818	-0.8931818182
60		3.93	-0.80	-100	4.644545455	-0.7145454545
80		3.93	-0.63	-78.75	4.465909091	-0.5359090909
100		3.93	-0.40	-50	4.287272727	-0.3572727273
140	SUPPORT	3.93	-	-	0	0
180		3.93	0.40	50	3.572727273	0.3572727273
200		3.93	0.60	75	3.394090909	0.5359090909
220		3.93	0.80	100	3.215454545	0.7145454545
240		3.93	1.00	125	3.036818182	0.8931818182
260		3.93	1.20	150	2.858181818	1.071818182
280		3.93	1.40	175	2.679545455	1.250454545
300		3.93	1.60	200	2.500909091	1.429090909
320		3.93	1.80	225	2.322272727	1.607727273

340		3.93	2.00	250	2.143636364	1.786363636
360		3.93	2.20	275	1.965	1.965
380		3.93	2.40	300	1.786363636	2.143636364
400		3.93	2.60	325	1.607727273	2.322272727
440	CUT	3.93	-	-	0	0
480		3.93	2.20	275	0.8931818182	3.036818182
500		3.93	1.80	225	0.7145454545	3.215454545
520		3.93	1.30	162.5	0.5359090909	3.394090909
540		3.93	0.90	112.5	0.3572727273	3.572727273
580	SUPPORT	3.93	-	-	0	0

Table 2. Comparison of Theoretical and Experimental Bending Moment

X (mm)	M _{expt} (N-mm)	M _{theo} (N-mm)	Error (Difference)	%Error	Ratio
0	-178.75	-175.06364	0.020838	2.105728	1.021057
20	-156.25	-150.05455	0.040453	4.128802	1.041288
40	-125	-125.04545	0.000364	0.036350	0.999636
60	-10	-100.03636	1.636484	90.003635	0.099964
80	-78.75	-75.02727	0.048417	4.961832	1.049618
100	-50	-50.01818	0.000364	0.036350	0.999636
140	-	0.00000	-	-	-
180	50	50.01818	0.000364	0.036350	0.999636
200	75	75.02727	0.000364	0.036350	0.999636
220	100	100.03636	0.000364	0.036350	0.999636
240	125	125.04545	0.000364	0.036350	0.999636
260	150	150.05455	0.000364	0.036350	0.999636
280	175	175.06364	0.000364	0.036350	0.999636
300	200	200.07273	0.000364	0.036350	0.999636
320	225	225.08182	0.000364	0.036350	0.999636
340	250	250.09091	0.000364	0.036350	0.999636
360	275	275.10000	0.000364	0.036350	0.999636
380	300	300.10909	0.000364	0.036350	0.999636
400	325	325.11818	0.000364	0.036350	0.999636
440	-	0.00000	-	-	-

480	275	267.95455	0.025952	-2.629347	1.026293
500	225	214.36364	0.048417	-4.961832	1.049618
520	162.5	160.77273	0.010686	-1.074357	1.010744
540	112.5	107.18182	0.048417	-4.961832	1.049618
580	-	0.00000	-	-	-

Table 3. Influence Lines of Theoretical and Experimental Bending Moments

Mtheo	Influence Line (theo)	Mexp	Influence Line (exp)
-175.06364	-44.54545455	-178.75	-45.48346056
-150.05455	-38.18181818	-156.25	-39.75826972
-125.04545	-31.81818182	-125	-31.80661578
-100.03636	-25.45454545	-100	-25.44529262
-75.02727	-19.09090909	-78.75	-20.03816794
-50.01818	-12.72727273	-50	-12.72264631
0.00000	0	-	0
50.01818	12.72727273	50	12.72264631
75.02727	19.09090909	75	19.08396947
100.03636	25.45454545	100	25.44529262
125.04545	31.81818182	125	31.80661578
150.05455	38.18181818	150	38.16793893
175.06364	44.54545455	175	44.52926209
200.07273	50.90909091	200	50.89058524
225.08182	57.27272727	225	57.2519084
250.09091	63.63636364	250	63.61323155
275.10000	70	275	69.97455471
300.10909	76.36363636	300	76.33587786
325.11818	82.72727273	325	82.69720102
0.00000	0	-	0
267.95455	68.18181818	275	69.97455471
214.36364	54.54545455	225	57.2519084
160.77273	40.90909091	162.5	41.34860051
107.18182	27.27272727	112.5	28.6259542
0.00000	0	-	0

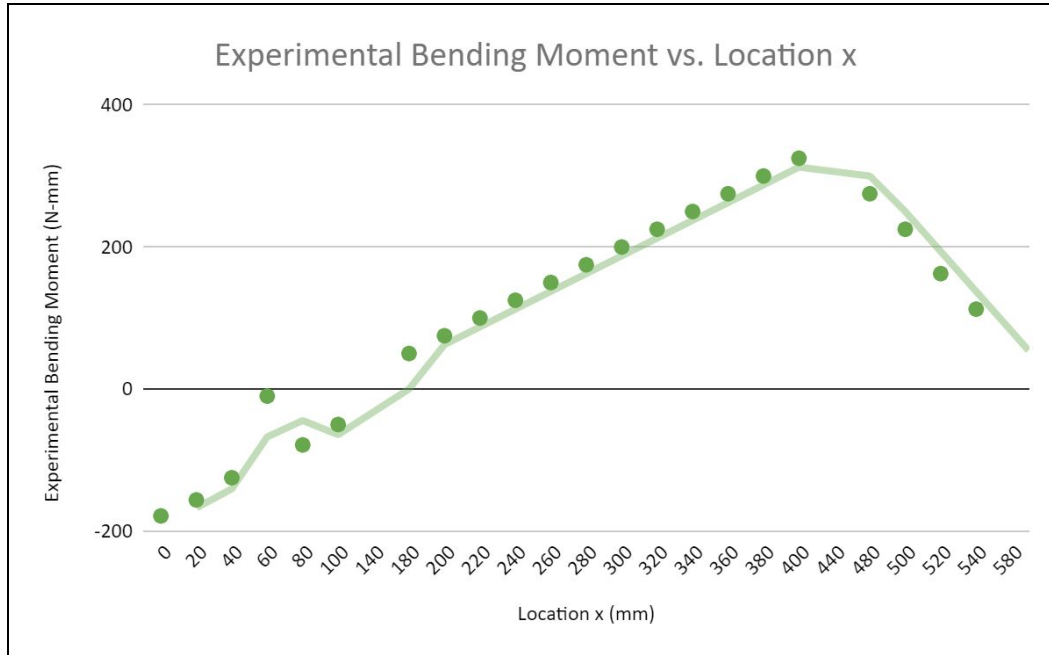


Figure 5. Experimental Bending Moment vs. Location x

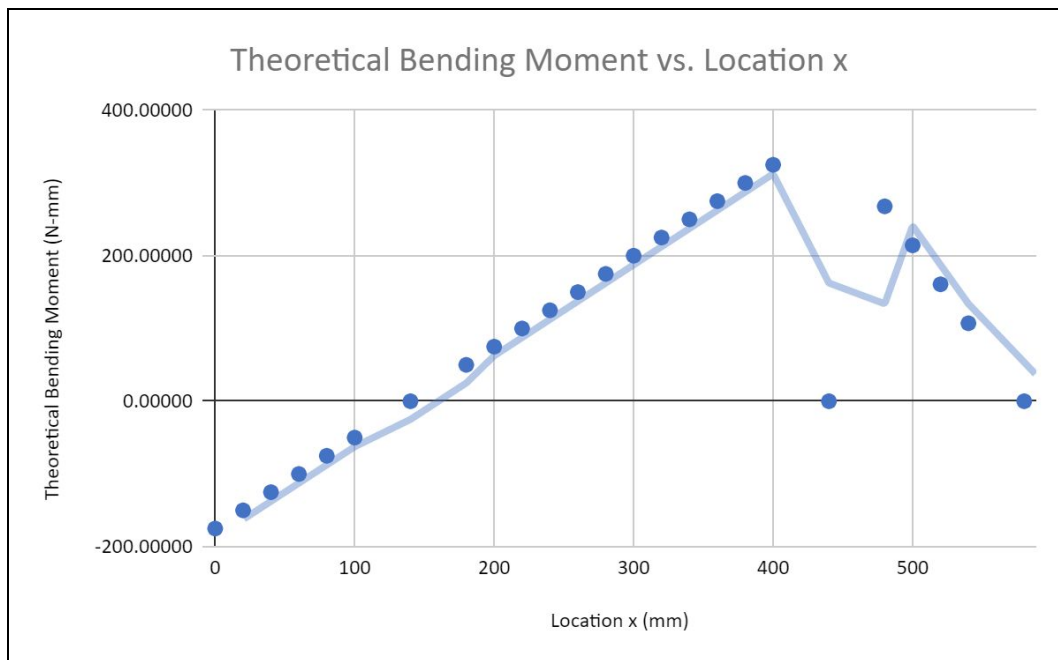


Figure 6. Theoretical Bending Moment vs. Location x

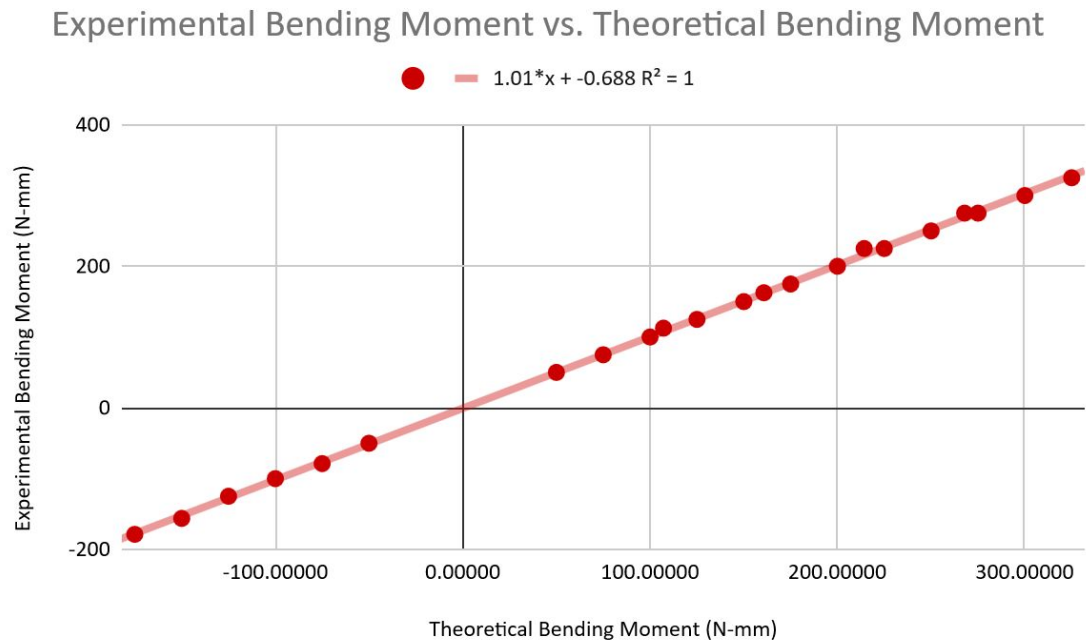


Figure 7. Experimental Bending Moment vs. Theoretical Bending Moment

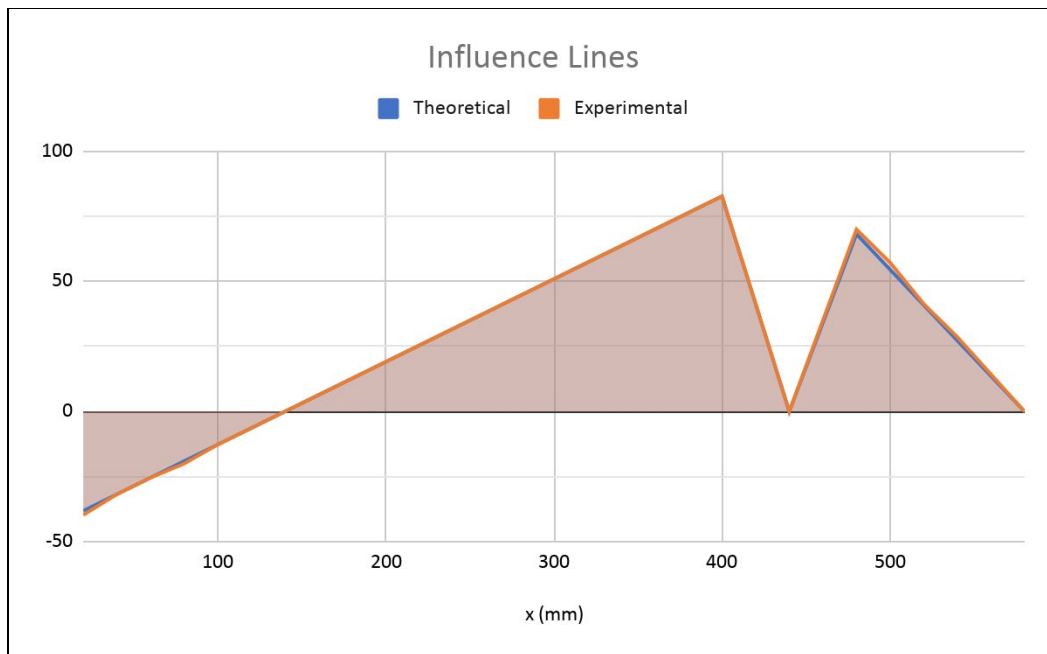


Figure 8. Comparison of Theoretical and Experimental Influence Lines

Table 4. Shear force data with Reactions at A and B

x	Description	P(N)	Force Data Average (N)	Ra	Rb
0		3.93	-1.10	5.180454545	-1.250454545
20		3.93	-0.95	5.001818182	-1.071818182
40		3.93	-0.80	4.823181818	-0.8931818182
60		3.93	-0.60	4.644545455	-0.7145454545
80		3.93	-0.45	4.465909091	-0.5359090909
100		3.93	-0.25	4.287272727	-0.3572727273
140	SUPPORT	3.93	-	0	0
180		3.93	0.45	3.572727273	0.3572727273
200		3.93	0.60	3.394090909	0.5359090909
220		3.93	0.80	3.215454545	0.7145454545
240		3.93	0.95	3.036818182	0.8931818182
260		3.93	1.10	2.858181818	1.071818182
280		3.93	1.30	2.679545455	1.250454545
300		3.93	1.45	2.500909091	1.429090909
320		3.93	1.60	2.322272727	1.607727273
340		3.93	1.80	2.143636364	1.786363636
360		3.93	1.90	1.965	1.965
380		3.93	2.10	1.786363636	2.143636364
400		3.93	2.30	1.607727273	2.322272727
440	CUT	3.93	-	0	0
480		3.93	-0.75	0.8931818182	3.036818182
500		3.93	-0.60	0.7145454545	3.215454545
520		3.93	-0.45	0.5359090909	3.394090909
540		3.93	-0.30	0.3572727273	3.572727273
580	SUPPORT	3.93	-	0	0

Sample Calculation for Reaction Forces:

$$R_A = P(580 \text{ mm} - x)/(580 \text{ mm} - 140 \text{ mm}); x = 0 \text{ mm} \text{ \& } P = 3.93 \text{ N}$$

$$R_A = (P(580 \text{ mm} - x))/(580 \text{ mm} - 140 \text{ mm}) \approx 5.180454545 \text{ N}$$

$$R_B = P - R_A$$

$$R_B = 3.93 \text{ N} - 5.180454545 \approx -1.250454545 \text{ N}$$

Table 5. Comparison of Theoretical and Experimental Shear Force

X (mm)	V _{exp}	V _{theo}	Error (Difference)	%Error	Ratio
0	1.10	1.250454545	0.1280216593	12.03198837	0.8796801163
20	0.95	1.071818182	0.1205035971	11.36556404	0.8863443596
40	0.80	0.8931818182	0.1100671141	10.43256997	0.8956743003
60	0.60	0.7145454545	0.1742738589	16.03053435	0.8396946565
80	0.45	0.5359090909	0.1742738589	16.03053435	0.8396946565
100	0.25	0.3572727273	0.3532934132	30.02544529	0.6997455471
140	0	0	0	0	0
180	-0.45	-0.3572727273	0.2297297297	25.95419847	1.259541985
200	-0.60	-0.5359090909	0.1128451381	11.95928753	1.119592875
220	-0.80	-0.7145454545	0.1128451381	11.95928753	1.119592875
240	-0.95	-0.8931818182	0.06165228113	6.361323155	1.063613232
260	-1.10	-1.071818182	0.02595228129	2.629346904	1.026293469
280	-1.30	-1.250454545	0.0388522545	3.962195565	1.039621956
300	-1.45	-1.429090909	0.01452478686	1.463104326	1.014631043
320	-1.60	-1.607727273	0.004817911294	0.4806333051	0.9951936669
340	-1.80	-1.786363636	0.007604562738	0.7633587786	1.007633588
360	-1.90	-1.965	0.03363518758	3.307888041	0.9669211196
380	-2.10	-2.143636364	0.0205655527	2.03562341	0.9796437659
400	-2.30	-2.322272727	0.009637132461	0.9590917988	0.990409082
440	0	0	0	0	0
480	0.75	0.8931818182	0.1742738589	16.03053435	0.8396946565
500	0.60	0.7145454545	0.1742738589	16.03053435	0.8396946565
520	0.45	0.5359090909	0.1742738589	16.03053435	0.8396946565
540	0.30	0.3572727273	0.1742738589	16.03053435	0.8396946565
580	0	0	0	0	0
			Average	9.274964504	0.8392920368

Sample Calculation for Shear Forces:

for $0 \text{ mm} \leq x \leq 440 \text{ mm}$, $V_{\text{(theo.)}} = R_2 - P$

$V_{\text{(theo.)}} = R_2 - P$; $x = 0 \text{ mm}$ & $P = 3.93 \text{ N}$

$V_{\text{(theo.)}} = 5.180454545 \text{ N} - 3.93 \text{ N} = 1.250454545 \text{ N}$

for $440 \text{ mm} < x \leq 580 \text{ mm}$, $V_{\text{(theo.)}} = R_2$

$$V_{\text{(theo.)}} = R_2; x = 480 \text{ mm} \ \& \ R_2 = 0.8931818182 \text{ N}$$

$$V_{\text{(theo.)}} = 0.8931818182 \text{ N} = 0.8931818182 \text{ N}$$



Figure 9. Experimental Shear Force vs. Location x

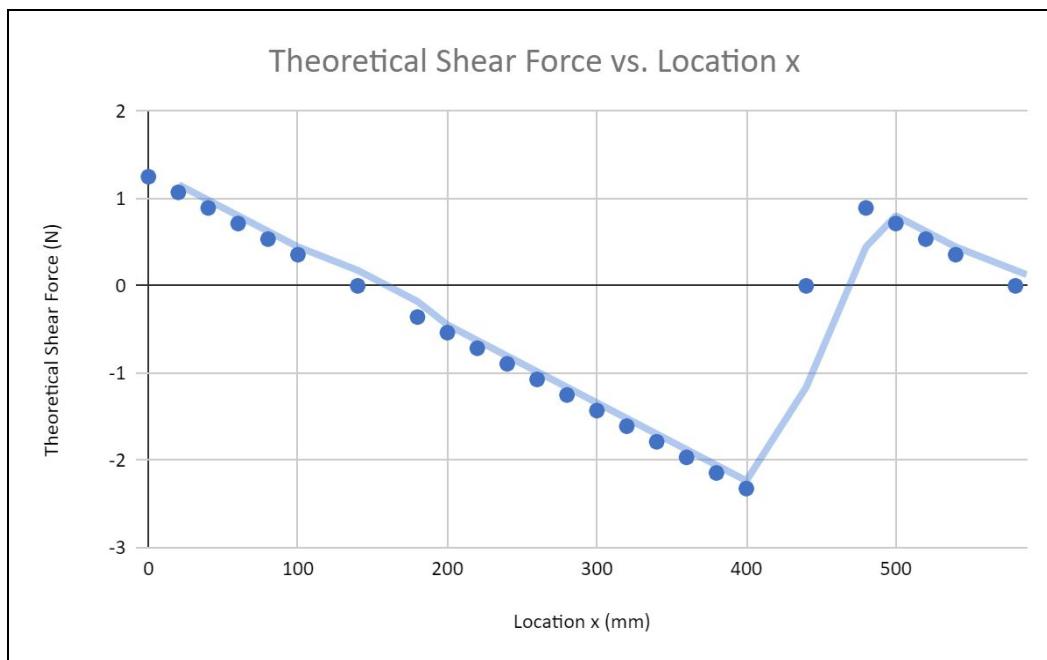


Figure 10. Theoretical Shear Force vs. Location x

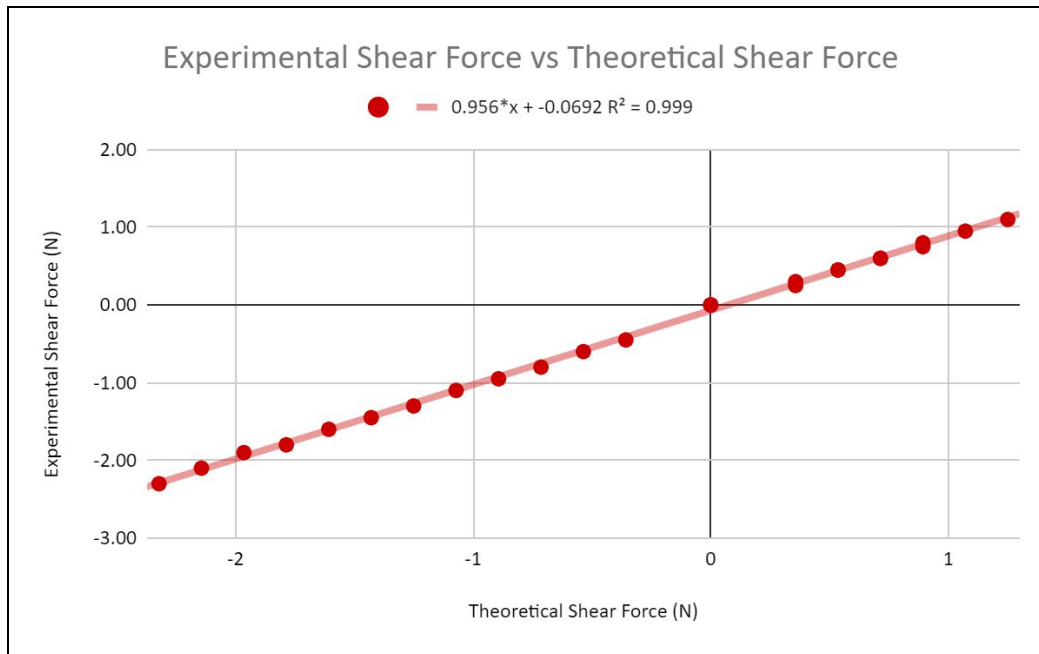


Figure 11. Experimental Shear Force vs. Theoretical Shear Force

IV. Theory and Calculation

This section provides a graphical representation used to visualize the concentrated load P of 3.93 Newtons at each designated location with a corresponding interval of 20 millimeters, and its ensuing reactions which aids in the analysis of the bending moment and shear force of the structure. It is essential to note that the bending moment and shear force at the cut must be proportional to the algebraic sum of the forces from the left or right of the cut of the structural model.

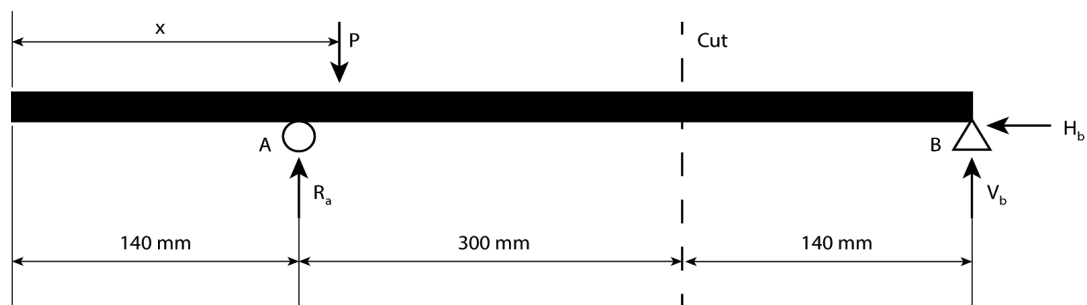


Figure 12. Free Body Diagram of the Structural Model

The experiment aims to analyze the bending moment and shear influence line at a cut dependent upon the live load and its distance from the left hand side of the beam which may be assessed through the STR-2 and STR-3 apparatus respectively. In deriving the theoretical values of the bending moment and shear force, the beam must be analyzed at only one section. Provided this, the equations would only consist of the known variables, which is the location (x) from 0 to

580 millimeters with a corresponding 20 millimeter interval, and the moving applied load (P), which obtains a value of 3.93 Newtons. To calculate the theoretical values, the vertical reaction at A may be computed by summing up the moments at B in terms of P and x and setting the moment in a clockwise positive direction. To identify the vertical reaction at B, take the summation of the forces found at the y-axis.

$$+ \square \Sigma M_B = 0 \quad (\text{eq. 1})$$

$$A_y(440) = P(580-x)$$

$$A_y(440) = 3.93(580-x)$$

$$A_y = \frac{3.93(580-x)}{440} \quad (\text{eq. 2})$$

$$+ \uparrow \Sigma F_y = 0 \quad (\text{eq. 3})$$

$$B_y + A_y = P$$

$$B_y = P - A_y$$

$$B_y = 3.93 - \frac{3.93(580-x)}{440} \quad (\text{eq. 4})$$

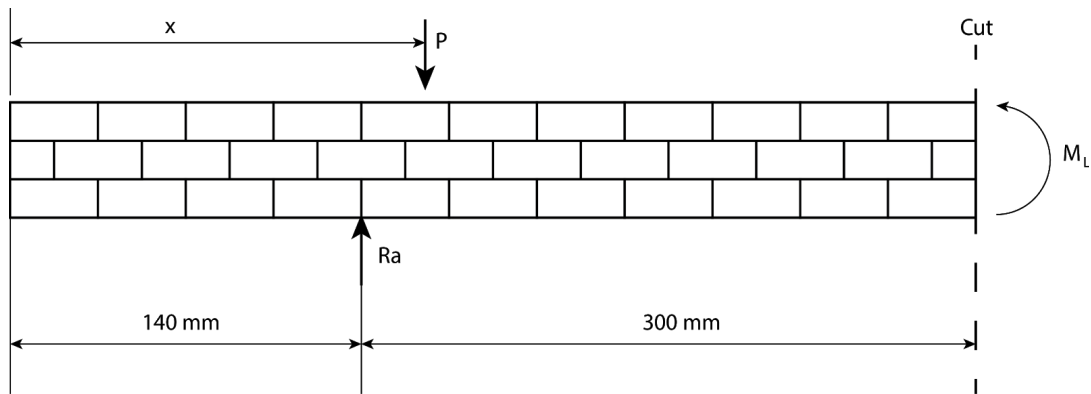


Figure 13. Moment Diagram of the Moving Concentrated Load at a Varied Distance (x) on the Left Hand Side of the Beam

From the overhanging beam, only one reaction is allowed to be incorporated in the analysis as there is one section to be examined. The left hand side is utilized to formulate an equation that will compute for the moment at the cut. From $x = 0$ mm to $x = 440$ mm, the section involves A_y , P and the moment, which has an assumed counterclockwise direction at the cut (as seen in Figure 8). Since A_y is already determined, the equation for the moment at the left hand side is as follows:

$$M_L = -A_y(300) + P(440-x)$$

$$M_L = A_y(300) - P(440-x)$$

$$M_L = A_y(300) - 3.93(440-x)$$

$$M_L = \frac{3.93(580-x)}{440} (300) - 3.93(440-x) \quad (\text{eq. 5})$$

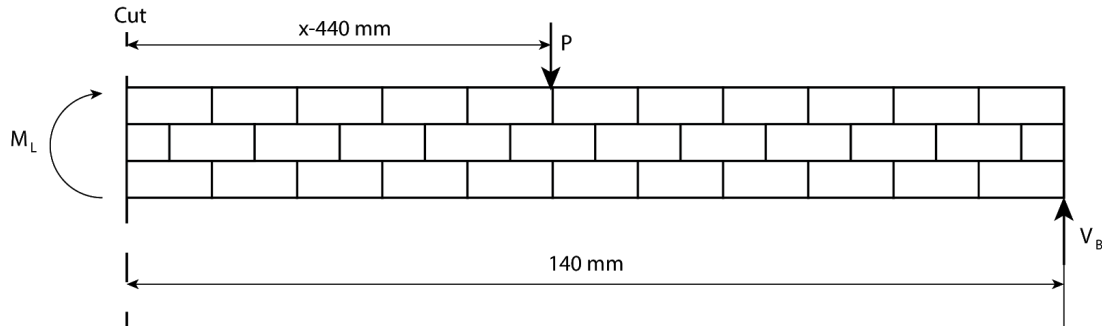


Figure 14. Moment Diagram of the Moving Concentrated Load at a Varied Distance (x) on the Right Hand Side of the Beam.

As the concentrated load passes the cut, which is at 440 millimeters from the left hand side of the beam, the left hand side no longer contains load P; therefore, the equation for the moment which is assumed to be in a clockwise direction from the cut with a distance between $x = 440\text{mm}$ to $x = 580\text{mm}$, considering the left hand side would be adjusted is as follows:

$$M_R = B_y(140)$$

$$M_R = (3.93 - \frac{3.93(580-x)}{440})(140) \quad (\text{eq. 6})$$

Derived from the structural model, the formulas below were employed for the theoretical moment at the cut:

$$\text{LHS: } M_L = \frac{3.93(580-x)}{440} (300) - 3.93(440-x)$$

$$\text{RHS: } M_R = (3.93 - \frac{3.93(580-x)}{440})(140)$$

In this scenario, the formulas reveal to be proportional at the left and right hand side of the beam wherein as a certain value of the distance from the concentrated load is substituted, both of the resulting moments of the left and right hand side would yield an equivalent value. Thus, it would be desirable to select a portion of the beam which involves less variable so that calculations would be conducted efficiently. For instance, when the distance is substituted to 20 millimeters, the moment for both left and right hand side of the beam is -150.05 N-mm (as seen in Table 2)

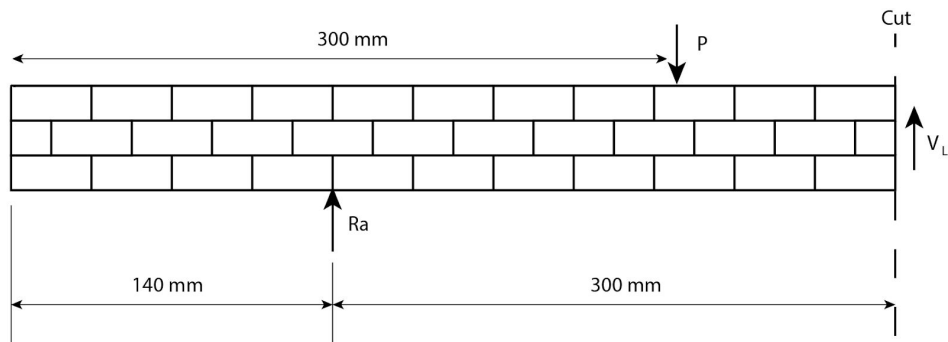


Figure 15. Shear Diagram of the Moving Concentrated Load at a Varied Distance (x) on the Left Hand Side of the Beam

As seen in Figure 16, the overhanging beam was analyzed on the left hand side. Provided this, the shear force at the cut section may be determined.

$$\begin{aligned}
 + \uparrow \Sigma F_y &= 0 \\
 V_L + A_y - P &= 0 \\
 V_L &= P - A_y \\
 V_L &= 3.93 - \frac{3.93(580-x)}{440} \quad (\text{eq. 7})
 \end{aligned}$$

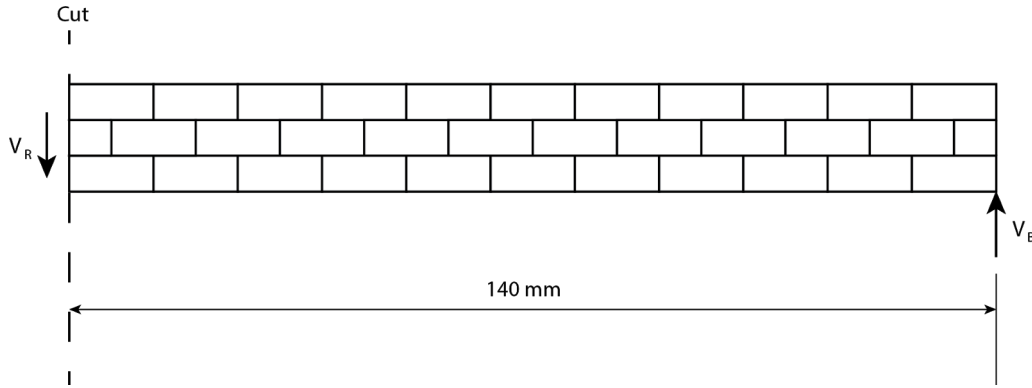


Figure 16. Shear Diagram of the Moving Concentrated Load at a Varied Distance (x) on the Right Hand Side of the Beam

As seen in Figure 16, the overhanging beam was analyzed on the left hand side. Provided this, the shear force at the cut section may be determined.

$$\begin{aligned}
 + \uparrow \Sigma F_y &= 0 \\
 B_y - V_R &= 0 \\
 V_R &= B_y \\
 V_R &= 3.93 - \frac{3.93(580-x)}{440} \quad (\text{eq. 8})
 \end{aligned}$$

Derived from the structural model, the formulas below were employed for the theoretical shear force at the cut—satisfying the aforementioned condition such that the value of the shear force at the left hand side must be proportional to the right hand side of the beam:

$$\begin{aligned}
 \text{LHS: } V_L &= 3.93 - \frac{3.93(580-x)}{440} \\
 \text{RHS: } V_R &= 3.93 - \frac{3.93(580-x)}{440}
 \end{aligned}$$

Using equations (7) and (8), the theoretical values for the shear at the cut may be computed, bearing in mind that equation (7) may only be used if the load occurs from $x = 0\text{mm}$ to $x = 440\text{mm}$, while equation (8) may be used from $x = 440\text{mm}$ to $x = 580\text{mm}$.

V. Data Analysis and Conclusions

Performing the necessary calculations, Table 2 shows the comparison between the experimental bending moment and the theoretical bending moment. The maximum negative and positive moments in the experimental data obtained were observed to be -178.75 N-mm at $x = 0$ mm and 325 N-mm at $x = 400$ mm, respectively. Similarly, the researchers also located the maximum positive and negative moments in the theoretical data at $x = 0$ mm and $x = 400$ mm, respectively, with bending moment values equal to -175.0636 N-mm and 325.1182 N-mm. Furthermore, the highest and lowest percent error computed is equal to approximately 90 percent and 0.036, respectively. The percent ratio was observed to range between 0.999636 and 1.0496 (see Table 2).

The theorem states that provided a live concentrated load, it must be positioned at a point on the influence line with the highest ordinate to obtain either the maximum positive or negative bending moment and shear force. In this scenario, the magnitude of the load must be multiplied to the ordinate to attain the maximum positive or negative value for moment and shear. With the aforementioned experimental and theoretical moment, both occurred at $x = 0$ mm from the left end which is the maximum negative bending moment, obtaining a value of -175.06364 N-mm. With a discrepancy of 0.2084 and a percentage error of 2.1057% which is considered as an acceptable value. The researchers deemed that the experimental data is consistent with the theory. Whereas, the maximum positive bending moment was located at $x = 400$ mm is 325.1182 N-mm with a discrepancy of 0.00036 and a percentage error of 0.03635. As the given values of error was identified, it may be inferred that both the maximum positive and negative bending moment is also consistent with the theory due to the resulting negligible difference and percentage error.

Figure 5 shows the plot of the experimental bending moment in N-mm with respect to the location x in millimeters. In this graph, a moving average trendline was utilized to fit the corresponding data points. Some data points are quite far away from the trendline, but overall, the chosen trendline fits all the data points. Figure 6 illustrates the theoretical bending moment with respect to the location x . Also, the researchers used a moving average trendline to fit the data points. Minor differences were reported between Figure 5 and Figure 6. This can be due to the random errors done in the experiment (i.e., computation, force sensor reading, weighing, etc.). Figure 7 illustrates the plot of the experimental bending moment with respect to the theoretical bending moment. In this graph, the researchers observed that a 45-degree line could fit most of the data points with a linear trendline equation of $1.01x - 0.688$ and a coefficient of determination of 0.9996. A high R^2 denotes that the regression line used by the researchers fits the observed data well.

Table 5 illustrates the comparison between the experimental shear and the theoretical shear. The maximum negative and positive shear, looking at the experimental data, is located at $x = 400$ mm, which has -2.30 N-mm and at $x = 0$ mm equal to 1.10 N-mm, respectively. On the other hand, the maximum negative and positive shear in the theoretical data is located at $x = 400$ mm and $x = 0$ mm with values equal to -2.322 N-mm and 1.250 N-mm, respectively. In this case, the maximum negative shear occurred at $x = 400$ mm from the left end with a value of -2.30 N-mm wherein its discrepancy is at around 0.0096 and a percentage error of 0.959%. This signifies that the experimental value is indeed consistent with the theory. On the other hand, the maximum positive shear occurs at $x = 0$ mm obtaining a theoretical value of 1.2504. Comparing the experimental shear force to the computed theoretical value, both occurred at $x = 0$ mm with a discrepancy of 0.12802 and a percentage error of 12.03%. Similarly, the researchers reckoned that

the experimental data is consistent with the theory due to the minimal calculated disparity for both the maximum positive and negative shear force.

Figure 9 and Figure 10 show the plot of the experimental shear force and theoretical shear force with respect to its location x . Similarly, a moving average trendline was also utilized to fit the corresponding data points. Most data points in both Figures 9 and 10 fell under the chosen trendline, and again, there were a few data points that were quite far from the regression line. This discrepancy can be attributed to different errors done in the experiment. Figure 11 shows the plot of the experimental shear force with respect to the theoretical shear force. A 45-degree line can fit all the data points, implying a strong linear relationship between the two variables (experimental and theoretical shear). Moreover, a trendline equation and coefficient of determination of $0.956x - 0.0692$ and 0.999 were observed, respectively. The graphs illustrate the highest ordinate at $x = 0\text{mm}$, $x = 400\text{mm}$, and $x = 480\text{mm}$ respectively (as seen in Figure 9 and 10). At any magnitude of the applied load, the shear at each location was computed by multiplying the ordinate of the influence line to the magnitude of the load.

Influence lines are essential in designing structures that would resist large live loads since influence lines are graphical representations of the response functions' variation, such as support reaction, shear and bending moment at a section, force in truss member as the downward unit load moves over the member. This method is utilized to assess a uniformly distributed live load and to analyze trusses. Comparing influence lines to shear and moment diagrams, influence lines represent the effect of a moving load at a specified point on a member. In contrast, the latter represents the effect of fixed loads at all points along with the member. Additionally, influence lines are useful in determining the deflections in a beam. With the aid of the influence line's ordinate, the magnitude and the character of the function would be obtained. Identifying the influence line may also help engineers in properly designing infrastructures such as highways and bridges to mitigate casualties and failures in the design since the aforementioned structures involve massive live loads such as trucks and other moving vehicles (MATHalino, c.a. 2020; University of Memphis, n.d).

Based on the theoretical and experimental diagrams for the influence line, both satisfied the qualitative analysis of a shear and moment influence line such that a roller must be placed at the cut or section that was considered while applying a negative shear or moment. In this case, the experimental influence line was valid as it concurred with the qualitative analysis as well as the theoretical influence line of the beam.

The possible contributors of errors to this experiment are the external factors. The external factors can be the change in the environment which includes the temperature and the humidity. Both can contribute to the minimal deviation of the theoretical and experimental data. These errors can be prevented when doing the experiment in a controlled environment. Having a controlled environment can help in achieving more accurate results that the researchers want. Another error that can also affect the experiment is the human error. These are mistakes such as wrong readings and wrong computations. This can be solved by double checking the works of the fellow researchers where all must contribute with what they know about the experiment. Lastly, the experiment can put out more accurate results by using the whole value of the computed solutions in solving the other parts of the experiment.

VI. References

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